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Flexible smart structures are large mechanical structures with applications in which specific performance characteristics are desired in the presence of parameter variations and disturbances. These structures tend to have severe cotnroller effort restrictions and tightly spaced lightly damped modes. When designing controllers for smart structures, H2 controllers are well suited for control effort restrictions and closed loop performance specifications while H infinite controllers are better suited for modeling uncertainties. This paper examines the application of H2/H infinity controller design methodologies to smart structures.

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ROBUST MULTIOBJECTIVE CONTROLLERS FOR SMART STRUCTURES

Vittal S. Rao,[1] Robert Butler,[1] Wei Zhao[2]

ABSTRACT Flexible smart structures are large mechanical structures with applications in which specific performance characteristics are desired in the presence of parameter variations and disturbances. These structures tend to have severe controller effort restrictions and tightly spaced lightly damped modes. When designing controllers for smart structures, H_2 controllers are well suited for control effort restrictions and closed loop performance specifications while H_∞ controllers are better suited for modeling uncertainties. This paper examines the application of H_2/H_∞ controller design methodologies to smart structures.

A unique feature of mechanical distributed systems is that state space systems can be determined from finite element models (FEM) in which the states have physical significance. For certain systems, these states can be directly measured by using a distributed PVDF film appropriately shaped and applied to the structure. This full state feedback system allows for the implementation of H_2/H_∞ full state feedback algorithms. The development of a control system implementing this type of algorithm is described for a simple cantilever beam.

In addition, a H_2/H_∞ algorithm developed by Bernstein and Haddad is investigated for the cantilever beam. This method does not require state measurement, but the controller design algorithm is slightly more complicated. This algorithm requires the solution of one algebraic Riccati equation and two coupled algebraic Riccati equations which are solved iteratively.

INTRODUCTION

Control of smart structures presents a number of difficult problems which arise due to structural parameter variations, limited actuation force, nonavailability of states for feedback and restrictive control bandwidth. Several robust control system design techniques have been developed and utilized in the design of controllers for smart structures (Lashlee et al., 1994; Damle et al., 1994; Lashlee, 1994). Amongst these robust control techniques, mixed norm H_2/H_∞ optimal control methods have many advantages for designing controllers for smart structures. These control techniques combine the good closed loop performance of H_2 controllers with the good stability robustness of H_∞ controllers. Mixed norm H_2/H_∞ controllers are less conservative for

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systems with known disturbance power spectral densities and poorly modeled disturbances and provide a trade-off between H_2 and H_∞ performance. Three different approaches for the formulation and solutions of H_2/H_∞ controllers are available in the literature (Bernstein and Haddad, 1989; Bernstein, Haddad and Nett, 1989; Doyle, Zhou and Bodenheimer, 1989; Khargonekar and Rotea, 1991). The application of H_2/H_∞ controller design methodologies for smart structures is reported in this paper. We also propose to develop distributed sensors for the measurement of physical quantities of structural systems. These measured quantities can be used for the implementation of H_2/H_∞ controllers.

The smart material polyvinylidene fluoride (PVDF) film has been utilized in the development of customized distributed sensors for simple cantilever beams. These measurements are used for the identification of mathematical models of the structure from experimental data. A simple direct state feedback controller has been demonstrated using the proposed distributed sensors.

MIXED H_2/H_∞ OPTIMAL CONTROL TECHNIQUES

Recently there has been a great deal of interest in the formulation and solution of a mixed H_2/H_∞ control methodology which can handle bounded spectrum and bounded energy inputs simultaneously. These methods promise robustness and performance in the controller design.

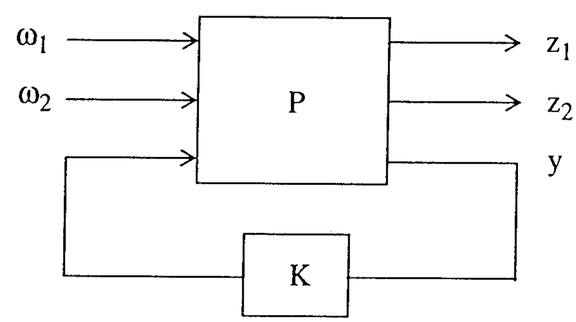

Figure 1. Standard Mixed Norm H_2/H_∞ Problem

The design objective is to find an admissible controller $K(s)$ in Figure (1) such that (i) $\|z_1\|_2$ is minimized in the face of ω_1 while at the same time (ii) $\|T_2\|_\infty < r$ holds, where T_2 is the transfer function between ω_2 to z_2. The infinity norm $\|T_2\|_\infty < r$ guarantees a given level of robustness, and the smaller value or r yields a better robust system. A trade off between the two design objectives has to be made in order to satisfy both of these.

A. BERNSTEIN AND HADDAD'S ALGORITHM

Among the three groups, Bernstein and Haddad were the earliest to work on the mixed norm H_2/H_∞ problem (Bernstein and Haddad, 1989; Bernststein, 1989). In their work, a special case of the general mixed norm H_2/H_∞ problem was considered, where only one exogenous input signal was used. That is, $\omega_1 = \omega_2 = \omega$, where the disturbance set ω is interpreted as a standard white noise signal.

Correspondingly, T_1 and T_2 are defined as transfer functions from ω to z_1 and ω to z_2 respectively. The nth order plant P in state space form is given by

$$P = \begin{bmatrix} A & D_1 & B \\ E_1 & 0 & E_2 \\ E_{1\infty} & 0 & E_{2\infty} \\ C & D_2 & 0 \end{bmatrix} \tag{1}$$

Alternatively, the problem can be formulated a follows:
Given a nth order plant

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) + D_1\omega(t) \\ y &= Cx(t) + D_2\omega(t) \end{aligned} \tag{2}$$

a nth order dynamic compensator given by

$$\begin{aligned} \dot{x}_c &= A_c x_c(t) + B_c y(t) \\ u(t) &= C_c x_c(t) \end{aligned} \tag{3}$$

can be determined such that it satisfies the performance criterion

$$J(A_c, B_c, C_c) = \lim_{t\to\infty} E\{x^T R_1 x + u^T R_2 u\} \tag{4}$$

with an H_∞ performance bound on the closed loop transfer function $H(s)$ from $\omega(t)$ to $E_{1\infty}x(t) + E_{2\infty}u(t)$ given by

$$\|H(s)\|_\infty \le \gamma \tag{5}$$

The compensator satisfying the conditions in Equations (4) and (5) is given by

$$\begin{aligned} A_c &= A - Q\Sigma - \Sigma PS + \gamma^{-2} QR_{1\infty} \\ B_c &= QC^T V_2^{-1} \\ C_c &= -R_2^{-1} B^T PS \end{aligned} \tag{6}$$

where Q, P and $\hat{Q}$ satisfy the coupled Riccati equations

$$0 = AQ + QA^T + V_1 + \gamma^{-2} QR_{1\infty} Q - Q\Sigma Q \tag{7}$$

$$0 = (A + \gamma^{-2}[Q + \hat{Q}]R_{1\infty})^T P + P(A + \gamma^{-2}[Q + \hat{Q}]R_{1\infty}) + R_1 - S^T P\Sigma PS \tag{8}$$

$$\begin{aligned} 0 = {} & (A - \gamma PS + \gamma^{-2} QR_{1\infty})\hat{Q} + \hat{Q}(A - \Sigma PS + \gamma^{-2} QR_{1\infty})^T \\ & + \gamma^{-2}\hat{Q}(R_{1\infty} + \beta^2 S^T P\Sigma PS)\hat{Q} + Q\Sigma Q \end{aligned} \tag{9}$$

The algorithm described above assumes the original system is a strictly proper system. Usually structural systems are described by proper transfer functions rather than strictly proper systems. Hence the Bernstein and Haddad algorithm has been modified to accommodate the proper transfer function system.

If we remove the restriction on the strictly proper system, P becomes

$$P = \begin{bmatrix} A & D_1 & B \\ E_1 & 0 & E_2 \\ E_{1\infty} & 0 & E_{2\infty} \\ C & D_2 & D \end{bmatrix} \tag{10}$$

Using the following solutions of the Riccati equations for Q, $\hat{Q}$ and P:

$$AQ + QA^T + -Q(\Sigma - \gamma^{-2}R_{1\infty})Q + V_1 = 0 \tag{11}$$

$$(A + \gamma^{-2}[Q + \hat{Q}]R_{1\infty})^T P + P(A + \gamma^{-2}[Q + \hat{Q}]R_{1\infty}) + R_1 - S^T P \Sigma P S = 0 \tag{12}$$

$$\begin{aligned} 0 = {} & (A + \gamma^{-2}QR_{1\infty} - \Sigma PS)\hat{Q} + \hat{Q}(A + \gamma^{-2}QR_{1\infty} - \Sigma PS)^T \\ & + Q\Sigma Q + \gamma^{-2}\hat{Q}\hat{R}_{1'\infty}\hat{Q} + \gamma^{-2}\beta^2\hat{Q}S^T P\Sigma PS\hat{Q} \end{aligned} \tag{13}$$

the controller (A_c, B_c, C_c) is obtained as

$$A_c = A - \Sigma PS - Q\Sigma + \gamma^{-2}QR_{1\infty} + QC^T V_2^{-1} D R_2^{-1} B^T PS \tag{14}$$

$$B_c = QC^T V_2^{-1} \tag{15}$$

$$C_c = -R_2^{-1} B^T PS \tag{16}$$

and the cost function J is computed by substituting $\tilde{q}$ and $\tilde{R}$.

$$J = tr\{(Q + \hat{Q})R_1 + \hat{Q}S^T P\Sigma PS\}. \tag{17}$$

B. KHARGONEKAR AND ROTEA'S ALGORITHM

The real mixed control problem (MCP) was considered first by Rotea and Khargonekar in (Khargonekar and Rotea, 1991) towards solving a special case of full state availability. That is, no upper bound to the two-norm is used and the real two-norm is minimized. Two sets of exogenous input signals and two sets of controlled outputs are allowed.

Let the state-space description of P be given by

$$\dot{x} = Ax(t) + B_1\omega_1 + B_2\omega_2 + Bu \tag{18}$$

$$z_1 = C_1 x + D_1 u \tag{19}$$

$$z_2 = C_2 x + D_2 u \tag{20}$$

$$y = x. \tag{21}$$

Two different problems are considered in this technique:
Problem A. Constrained optimal control problem: For the plant P, find and admissible (internally stabilizable) controller K that achieves

$$inf\{\|T_1\|_2\text{: K admissible and } \|T_2\|_\infty < \gamma\}$$

where

$$T_1 = \frac{z_1}{\omega_1} \tag{22}$$

$$T_2 = \frac{z_2}{\omega_2} \tag{23}$$

Problem B. Find an admissible controller K that achieves

$$inf\{\|T_1\|_2\text{: K admissible}\}$$

and such that $\|T_2\|_\infty < \gamma$.

Note that Problem A represents a constrained optimization problem (with the constraint $\|T_2\|_\infty < \gamma$), whereas Problem B represents an unconstrained problem of minimizing an H_2-performance measure that also satisfies an H_∞-norm bound. If an admissible $K(s)$ exists and solves Problem B, then this $K(s)$ is also a solution to Problem A. Thus Problem B provides sufficient conditions for the existence of a solution to Problem A. The solution method minimizes an upper bound of the two-norm, but not the two-norm itself. The solution of this problem requires full state feedback. The distributed sensor proposed in the next section will provide the measurement of states for the implementation of H_2/H_∞ control.

DISTRIBUTED SENSORS FOR STATE MEASUREMENT

The measurement of the model states of a structural systems is possible by utilizing appropriately shaped PVDF film. PVDF film is a piezoelectric material available in thin sheets which can easily be cut to the desired shape. The charge developed on PVDF film is given by

$$q(t) = K_f \int_{l_1}^{l_2} b(x) \frac{\partial^2}{\partial x^2} y(x,t)\, \partial x \tag{24}$$

where K_f is a constant which is a function of the film and structure parameters and $b(x)$ is the shape of the sensor. The current generated by the film is proportional to the derivative of the charge since the film is a capacitive device. For the cantilever beam investigated in this paper, the parameters we are interested in measuring are the

displacement, rotation, velocity and angular velocity and discrete points on the structure. The shapes required for the point measurement of these quantities are derived in (Butler and Rao, 1994) and are very simple geometric shapes. This allows for the implementation of several sensors with a single sheet of film.

The measurement of point displacement is accomplished by measuring the charge from a triangularly shaped PVDF film sensor. The sensor segment labeled S2 in Figure (2) demonstrates the shape of the sensor for the measurement of the displacement of the beam at position L/2. The geometric sum of sensor segments S2, S3 and S5 show the shape for the measurement of the tip displacement. The rotation at a point on the cantilever beam is measured by taking the charge from a rectangularly shaped PVDF film. Sensor segments S1, S2 and S3 show the shape of the sensor for the measurement of the rotation of the beam at L/2. The summation of the charge from all five of the segments results in the rotation of the tip of the structure.

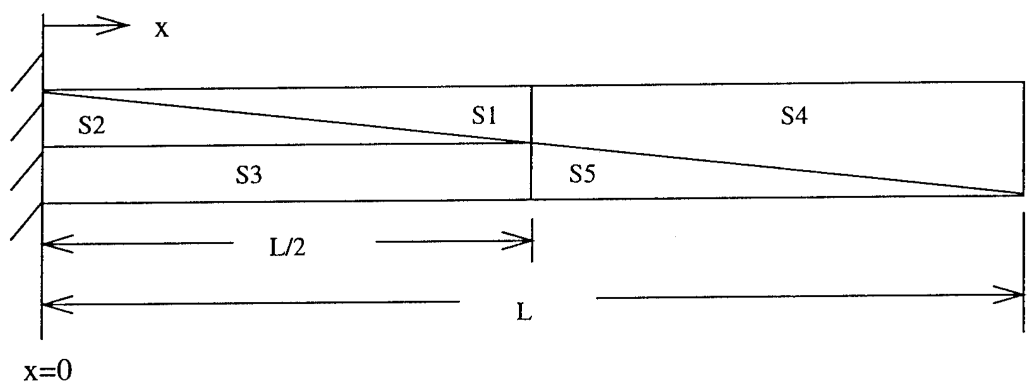

Figure 2. Segmented PVDF Film on Cantilever Beam

The measurement of the point velocity of the structure is accomplished by measuring the current on a PVDF film sensor with the same shape as the displacement sensor described above. The measurement of the point rotation rate is made be measuring the current from a sensor with the same shape as the rotation sensor described above. As shown in Figure (2), several sensor shape functions can be obtained by partitioning a single sheet of PVDF film into electrically isolated segments. A summary of the quantities that can be measured with the segmented PVDF sensor shown in Figure (2) are listed in Table I.

TABLE 1. Sensor Signals in Figure (2)

Physical Parameter	Segment Numbers	Measurement Type
Displacement at L/2	S2	Charge
Rotation at L/2	S1+S2+S3	Charge
Velocity at L/2	S2	Current
Angular Velocity at L/2	S1+S2+S3	Current
Displacement at L	S2+S3+S5	Charge
Rotation at L	S1+S2+S3+S4+S5	Charge
Velocity at L	S2+S3+S5	Current
Angular Velocity at L	S1+S2+S3+S4+S5	Current

SYSTEM IDENTIFICATION USING POINT MEASUREMENTS

In addition to simplifying controller implementation, the point measurements described in the previous section aid in the experimental system identification of the structure. This section describes a method which has been developed for the experimental identification of a state space model of a cantilever beam given by

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{25}$$

Models resulting from this system identification procedure closely resemble the finite element model of the structure and have states with the same physical significance. The states of the experimental model are displacements, rotations, velocities and angular velocities. Unlike the finite element model, it is not necessaries to generate a large model (several elements) in order to accurately represent a few modes.

Identification of the cantilever beam model begins by measuring the eigenvalues of the system that we are interested in modeling. In general, this can be done with swept sine measurements or by simply tuning the sinusoidal input frequency of the actuation signal to achieve the maximum observed response on any of the sensors. With the frequencies ω_d known, the damping of the mode can be measured by turning off the actuator once the steady state response is achieved and measuring the decay of the time domain signal. Performing these measurements for each of the modes of interest allows for the calculation of the matrix $\tilde{A}$ given by

$$\tilde{A} = T^{-1}AT, \tilde{B} = T^{-1}B, \tilde{C} = CT \tag{26}$$

where $\tilde{A}$ is the diagonalized system matrix. The linear transformation in Equation (26) given by $T = \begin{bmatrix} P_1 \ P_2 \ \dots \ P_n \end{bmatrix}$ consists of columns of eigenvectors P_i of the system.

The eigenvectors of the system matrix can be measured using a sinusoidal input signal at the frequency of the eigenvalue and measuring the steady state magnitude and phase of each of the distributed sensors. These quantities represent the eigenvectors P_i of the system and completely define the linear transformation to the diagonal system. For the cantilever beam with no significant actuator dynamics, all of the measurements will be in phase or 90° out of phase with the input signal. The system A matrix can be calculated by

$$A = T\tilde{A}T^{-1} \tag{27}$$

The $\tilde{B}$ matrix in Equation (26) of the diagonal system has the form

$$\tilde{B} = [b_1 j \quad -b_1 j \quad b_2 j \quad -b_2 j\,]^T \tag{28}$$

so that the state equation for a fourth order diagonal system is

$$\dot{\tilde{x}}_i = \tilde{A}\tilde{x}_i + \tilde{B}u_i = \tilde{A}\tilde{x}_i + \begin{bmatrix} b_1 j \\ -b_1 j \\ b_2 j \\ -b_2 j \end{bmatrix} u_i \tag{29}$$

where $\underset{\sim}{u}_i$ is the input corresponding to the ith mode. The elements b_i of the diagonal matrix $\tilde{B}$ can be calculated from the data collected for the identification of the A matrix. Solving Equation (29) for $\tilde{B}$ with the steady state sinusoidal input $u_i = q_i \cos(\omega_i t)$ gives

$$b_i = \frac{\dot{\tilde{x}}_i - \tilde{A}\tilde{x}_i}{q_i} = \frac{T^{-1}\dot{x}_i - T^{-1}Ax_i}{q_i}. \tag{30}$$

Since the vector x consists of sinusoids, $\dot{x}$ can be calculated. The time derivative of the measured states is given by

$$\dot{x}_i = \omega x_i j \tag{31}$$

yielding

$$b_i = \frac{\omega_i j T^{-1} x_i - T^{-1}Ax_i}{q_i} \tag{32}$$

Solving Equation (32) for each of the individual eigenvalue inputs gives the $\tilde{B}$ matrix. The B matrix can be determined using Equation (26).

Since the physical significance of the states is well known for this system, the C matrix can be selected as desired.

CANTILEVER BEAM FULL STATE FEEDBACK DESIGN EXAMPLE

The measurement of the states of a structural system allows for the implementation of full state feedback controllers such as pole placement, LQR, H_∞ and H_2/H_∞ controllers using only simple analog gain circuits. The linear quadratic regulator (LQR) controller has good robustness properties and will be used here to illustrate the simplicity of the full state controller implementation. The measurement of the states of the system eliminates the need for state estimation. This means that the robustness properties inherent in the LQR controller are retained in the controller implementation and do not need to be recovered with a loop transfer recovery methodology.

A finite element model of the structure shown in Figure (3) will be utilized to illustrate the benefits of the controller. As mentioned in the previous section, an experimental model can be determined for the actual system which accurately models all of the modes present in the model. The error inherent in finite element model using only one segment would not be present in the experimentally determined model.

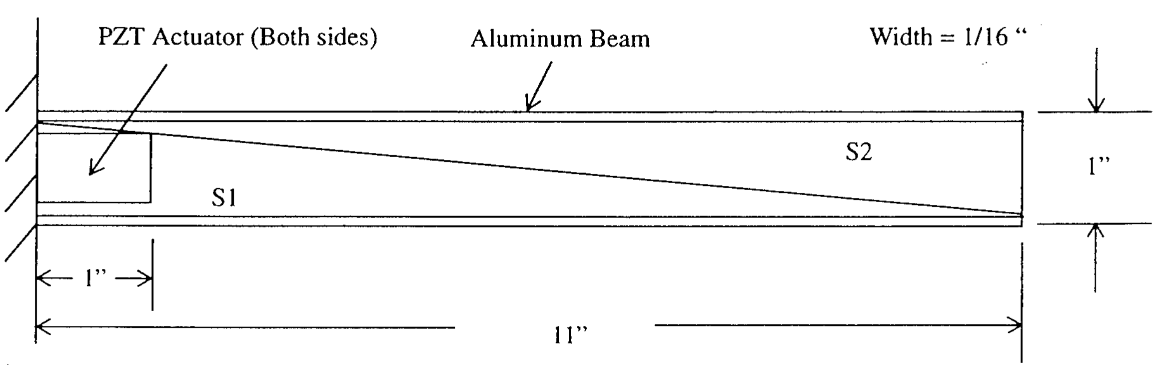

Figure 3. Cantilever Beam with Segmented PVDF Film Sensor

The distributed sensor shown in Figure (3) will provide the measurement of the tip displacement (charge on S1), rotation (charge on S1+S2), velocity (current from S1) and angular velocity (current from S1+S2). The mass and stiffness matrix for a corresponding single element finite element model of the beam is given by

$$m = \begin{bmatrix} 1.132e-2 & -4.459e-4 \\ -4.459e-4 & 2.265e-5 \end{bmatrix}, k = \begin{bmatrix} 3.261e2 & -4.556e1 \\ -4.556e1 & 8.486e0 \end{bmatrix} \tag{33}$$

The FEM state space model with 1% damping on each of the modes is given by

$$A = \begin{bmatrix} 1.01e1 & -4.18e1 & 1e0 & 0 \\ 5.62e1 & -1.24e1 & 0 & 1e0 \\ 2.25e5 & -4.78e4 & 1.01e0 & -4.18e-1 \\ 6.44e6 & -1.32e6 & 5.62e1 & -1.25e1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ -5.18e2 \\ -1.71e4 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{34}$$

In the LQR controller, the cost function

$$J = \int_0^{\infty} (x^T Q x + u^T R u)\, dt \tag{35}$$

is minimized subject to the constraint equation $\dot{x} = Ax + Bu$ The cost on the states and the cost on the control signal, Q and R respectively, were chosen to provide 11% damping on each of the modes.

$$Q = \begin{bmatrix} 2.11e7 & -4.03e6 & 0 & 0 \\ -4.03e6 & 8.08e5 & 0 & 0 \\ 0 & 0 & 1.50e2 & -8.51e0 \\ 0 & 0 & -8.51e0 & 9.23e-1 \end{bmatrix}, R = 30000 \tag{36}$$

resulting in the feedback gain matrix

$$K = \begin{bmatrix} 9.3e-1 & -1.73e-1 & 1.05e-1 & -8.57e-3 \end{bmatrix} \tag{37}$$

Simulations of the open and closed loop response is shown in Figure (4).

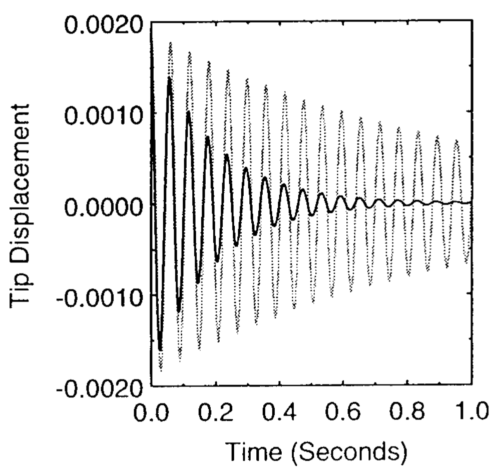

Figure 4. Simulated Open and Closed Loop Response of Cantilever Beam

CONCLUSIONS

A method to utilize multiobjective robust controllers for smart structures is presented in this paper. The proposed method provides a trade-off between closed-loop performance and disturbance attenuation. A simple modification is provided for designing controllers for systems with proper transfer functions.

We have also developed distributed sensor shape functions for the measurement of physical quantities of structural systems using PVDF film sensors. These sensor measurements can be used for the implementation of full state feedback H_2/H_∞ controllers on smart structures. We have demonstrated the applicability of distributed sensors for the implementation of full state feedback systems. We are currently working on implementation of these algorithms on test articles. We have also successfully demonstrated a procedure for system identification using point measurements.

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